Multi-target filtering with linearized complexity
Supplementary materials

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***Abstract*—These supplementary materials provide the implementation of the hypothesised filter for independent stochastic populations (HISP filter) under a different approximation, together with a numerical comparison with the approximation used in the main body of the article.**

I. Alternative approximation

We recall that for any index $\boldsymbol{k}$ in the set $\mathbb{I}_{t|t-1}$ of predicted hypotheses at time t, any resolution cell $z \in Z'_t$, and any likelihood ℓ^s_t with s in the union of Z'_t with the symbol d corresponding to the detection of targets, we denote by $p^{\boldsymbol{k},s}_t(z)$ the marginal likelihood for the observation of the hypothesis $\boldsymbol{k}$ in the resolution cell z. This marginal likelihood is also extended to $\bar{p}^{\boldsymbol{k},s}_t(z) = p^{\boldsymbol{k},s}_t(z) + L^s_t(z \,|\, \varphi)\bar{p}^{\boldsymbol{k}}_{t|t-1}(\varphi)$ to cover for the case where the hypothesis is erroneous, noting that the added term is null when z is different from the empty observation ϕ (since erroneous hypotheses cannot be observed). We additionally recall that the subscripts m, u and $\flat$ refer respectively to the previously-detected hypotheses, to the never-detected hypotheses and to the false alarm.

A. Statement

The two approximations stated in the article are as follows: for all $\boldsymbol{k} \in I$ with $I \subseteq \mathbb{I}^{\mathrm{m}}_{t|t-1}$ and for all $z \in Z$ with $Z \subseteq Z_t$, we assume that

A.3 $p^{\boldsymbol{k},\mathrm{d}}_t(z)p^{\boldsymbol{k}',\mathrm{d}}_t(z) \approx 0$ for any $\boldsymbol{k}' \in I$ such that $\boldsymbol{k} \neq \boldsymbol{k}'$, or

A.4 $p^{\boldsymbol{k},\mathrm{d}}_t(z)p^{\boldsymbol{k},\mathrm{d}}_t(z') \approx 0$ for any $z' \in Z$ such that $z \neq z'$.

The expression of the term $w^{\boldsymbol{k},z}_{\mathrm{ex}}$, central to the update step of the HISP filter, is shown to be greatly simplified under **A.3** in Theorem 3. A similar approach can be used to simplify the expression of $w^{\boldsymbol{k},z}_{\mathrm{ex}}$ under **A.4**.

Corollary 1. *For any $\boldsymbol{k} \in \mathbb{I}_{t|t-1}$ and any $z \in \bar{Z}_t$, applying **A.4** to the subsets $\mathbb{I}^{\boldsymbol{k}}_{t|t-1}$ and $Z_t \setminus \{z\}$, the scalar $w^{\boldsymbol{k},z}_{\mathrm{ex}}$ can be factorised as follows*

$$w^{\boldsymbol{k},z}_{\mathrm{ex}} = C''_t(\boldsymbol{k}, z) \prod_{z' \in Z_t \setminus \{z\}} \Bigg[C^{\mathrm{u},\flat}_t(z') + \sum_{\boldsymbol{k}' \in \mathbb{I}^{\mathrm{m}}_{t|t-1} \setminus \{\boldsymbol{k}\}} \frac{p^{\boldsymbol{k}',\mathrm{d}}_{t|t-1}(z')}{\bar{p}^{\boldsymbol{k}',\mathrm{d}}_{t|t-1}(\phi)} \Bigg]$$

where

$$C^{\mathrm{u},\flat}_t(z) = \frac{p^{\mathrm{u},\mathrm{d}}_t(z)}{\bar{p}^{\mathrm{u},\mathrm{d}}_t(\phi)} + \frac{p^{\flat,z}_t(z)}{\bar{p}^{\flat,z}_t(\phi)},$$

and where

$$C''_t(\boldsymbol{k}, z) = \big[\bar{p}^{\mathrm{u},\mathrm{d}}_t(\phi)\big]^{n^{\mathrm{u}}_t - \delta_{\mathrm{u}}(\boldsymbol{k})} \times \Bigg[\prod_{z \in Z'_t \setminus Z_\flat} \bar{p}^{\flat,z}_t(\phi) \Bigg] \Bigg[\prod_{\boldsymbol{k}' \in \mathbb{I}^{\mathrm{m}}_{t|t-1} \setminus \{\boldsymbol{k}\}} \bar{p}^{\boldsymbol{k}',\mathrm{d}}_{t|t-1}(\phi) \Bigg]$$

with $Z_\flat$ equal to $\{z\}$ when $\boldsymbol{k} = \flat$ and $\emptyset$ otherwise.

The term $C^{\mathrm{u},\flat}_t$ is the same as the one introduced in the article and is simply recalled here for convenience.

II. Performance assessment

The performance of the HISP filter for each available approximation is assessed on the same 100 Monte Carlo (MC) runs as in the article. The Optimal Sub-Pattern Assignement (OSPA) distance is shown in fig. 1 where it appears that the performance is the same for both approximations when the easier scenario of Case 1 is considered. However, there is a difference in performance for the more challenging scenarios of Cases 2 and 3, with Approximation **A.4** slightly outperforming **A.3** in Case 2 and with Approximation **A.3** slightly outperforming **A.4** in Case 3. This observation is consistent with the nature of the considered approximations since **A.3** assumes that two hypotheses are unlikely to produce a large marginal likelihood with the same observation, which is more likely to hold in a scenario with a high probability of detection, whereas **A.4** assumes that a given hypothesis is unlikely to produce a large marginal likelihood for two different observations, which is more likely to hold in a scenario with a low probability of detection.

This simulation study shows that the performance of the HISP filter is not overly dependent on the choice of the approximation and that the behaviour of the filter for different approximations is consistent with their intuitive interpretation.

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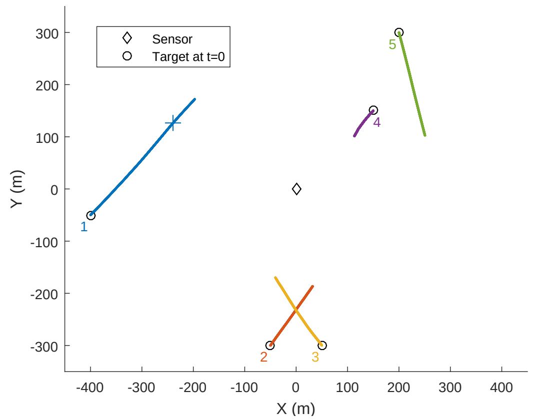
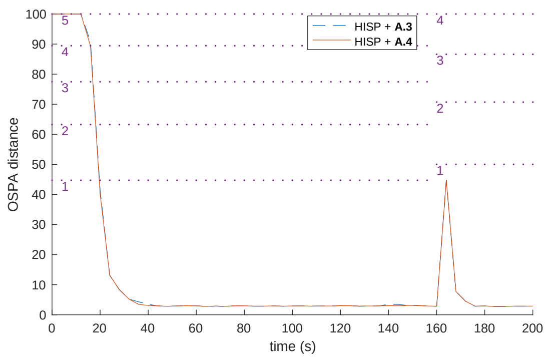
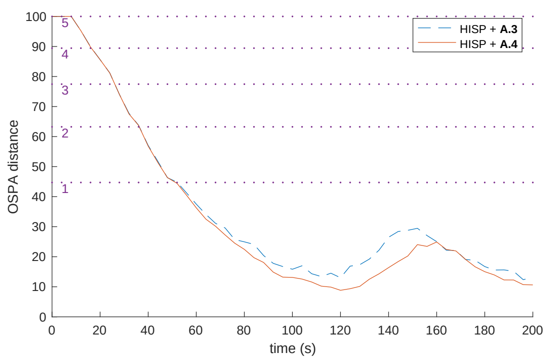


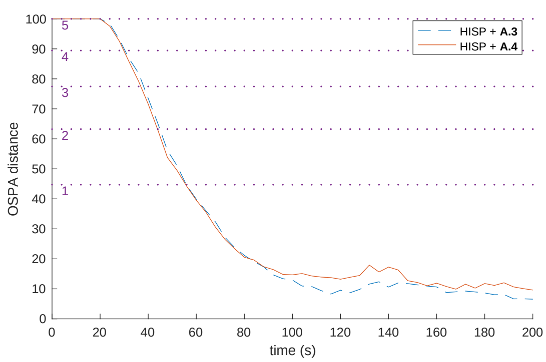
(a) A realisation of the target trajectories (blue cross: location of Target 1 when it disappears in Case 1.)



(b) Case 1: $p_{\rm d} = 0.995$ and $n_\flat \approx 83$.



(c) Case 2: $p_{\rm d} = 0.5$ and $n_\flat \approx 15$.



(d) Case 3: $p_{\rm d} = 0.8$ and $n_\flat \approx 167$.

Fig. 1: OSPA distance in Cases 1-3 (b-d) on the scenario (a) over 100 MC runs. (HISP filter with **A.4**: solid line – HISP filter with **A.3**: dashed line – the dotted line numbered n represents the OSPA for a cardinality-only error of n)