Exercise sheet 4
Solutions

Exercise 1. i) The noise $\boldsymbol{u}_k$ has distribution $\mathrm{N}(\cdot\,; 0, U_k + \tilde{U}_k)$

ii) Using the result from the lecture notes in the case of a one-dimensional state, we have

$$M_k = \frac{\sqrt{\breve{P}_k}}{\sqrt{P_k}} = \frac{\sqrt{P_k + \tilde{U}_k}}{\sqrt{P_k}} = \sqrt{1 + \tilde{U}_k / P_k}$$

so that

$$\begin{aligned} \breve{F}_k &= M_k F_k = M_k = \sqrt{1 + \tilde{U}_k / P_k} \\ c_k &= \breve{m}_k - M_k F_k \hat{m}_{k-1} = (1 - \breve{F}_k)\hat{m}_{k-1} \\ \breve{U}_k &= M_k U_k M_k^{\intercal} = (1 + \tilde{U}_k / P_k) U_k, \end{aligned}$$

as required.

Exercise 2. The intervention which consists of adding an independent noise term (resulting in the total covariance of the transition noise being $U_k + \tilde{U}_k$) cannot be used if the desired predicted variance is "smaller" than the original predicted variance P_k. Indeed, $P_k + \tilde{U}_k < P_k$ cannot hold since $\tilde{U}_k$ is a variance and is therefore positive definite. In this sense, the change-of-moments intervention is more general. In the context of a first-order polynomial trend model, this situation corresponds to $M_k < 1$ and, thus, $\breve{F}_k < 1$, c_k is of the same sign as $\hat{m}_{k-1}$ and $\breve{U}_k < U_k$.